Test 7 : Wednesday 7th September

Interval Estimates for Proportions

This assessment contributes 7% towards the final year mark.

40 minutes are allocated for this test.

Calculators are required.

No notes of ANY nature are permitted.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name : ANSWERS

40 minutes

Total = $\overline{38}$

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Question 1 **(3 marks)**

A law firm has 16 partners, 28 secretaries, 27 paralegals and 29 associates. How many people from each category do you need for a stratified sample of 20 people?

Partners $= \frac{16}{100} \times 20 = 3.2 \approx 3$ Secretaries $= \frac{28}{100} \times 20 = 5.6 \approx 6$ Paralegals $= \frac{27}{100} \times 20 = 5.4 \approx 5$ Associates $= \frac{29}{100} \times 20 = 5.8 \approx 6$ ∴ they should choose 3 partners, 6 secretaries, 5 paralegals and 6 associates.
✓ Calculates proportions for each category ✓ Multiplies proportions by sample size 20 ✓ Correctly rounds to give whole number answer

Question 2 **(3 marks)**

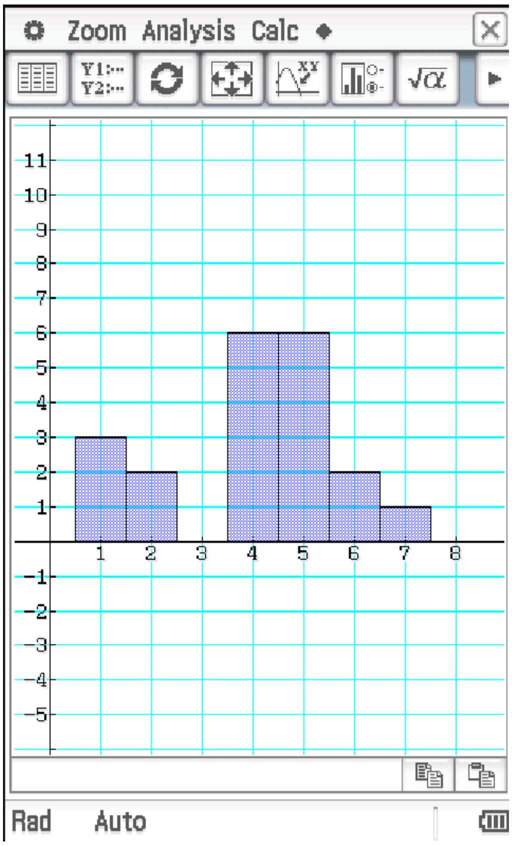
James, a Year 12 student, needs to construct a survey of which modes of transport are used by students at his school for an assignment. He decides to hand out his survey to other students in Year 12. Explain why this method will result in a sample that is not representative of the population and provide an alternative method for James to use.

Not a random sample as he is only surveying Year 12 students, which can lead to bias in the sample. A better method would be to assign each student in the school a number and use a random number generator to select which students to survey.
✓ Comments that it is not random and/or could be biased ✓ Gives a valid reason why it isn't random and/or could be biased ✓ Provides a valid alternative method for James to use

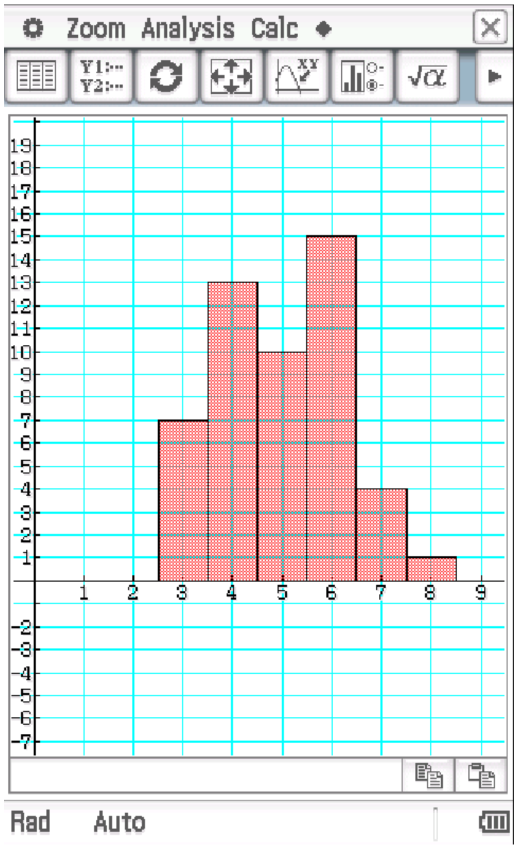
Question 3 **(11 marks)**

The two graphs on the calculator screen shots below display the results of two different simulations measuring the number of heads shown when a coin is tossed 10 times.

Simulation 1:



Simulation 2:



(a) State the probability distribution and the parameters for this situation.

(2 marks)

$X \sim Bin(10,0.5)$
✓ Recognises that the situation is binomial ✓ Gives correct n and p values

(b) What is the theoretical expected value and standard deviation for this situation?

(2 marks)

$Expected\ value = np = 10 \times 0.5 = 5$ $Std\ dev = \sqrt{npq} = \sqrt{10 \times 0.5 \times 0.5} = 1.58$
✓ Correctly calculates expected value ✓ Correctly calculates standard deviation

Question 3 (Continued)

(c) In Simulation 1 the mean value is 4 and the standard deviation is 1.70 while in Simulation 2 the mean value is 4.98 and the standard deviation is 1.27. By comparing these values with the answers to part (b) and the shape of the two histograms, explain how these simulations illustrate an important principle when collecting samples.

(3 marks)

Simulation 2 is more representative of the population with a very similar mean to the theoretical value and a better shape. This is to be expected as it has more trials in the sample. This shows that a sample will be more representative of the population distribution as the sample size increases.
✓ Correctly compares one aspect (mean/standard deviation/shape/sample size) for each simulation to that of the population distribution. ✓ Correctly compares two or more aspects (mean/standard deviation/shape/sample size) for each simulation to that of the population distribution. ✓ Describes link between sample size and the normality of the distribution.

(d) A third simulation is conducted with 100 trials of tossing a coin 10 times. If $\hat{p}$ represents the proportion of trials containing at least 6 heads, determine a probability density distribution and its parameters for $\hat{p}$.

(4 marks)

$X \sim Bin(10,0.5), P(X \geq 6) = 0.37695$ As sample size $n = 100 > 30$ $\hat{p}$ is approximately normally distributed. $\therefore\ \hat{p} \sim N\left(0.3770, \sqrt{\frac{0.3770 \times (1 - 0.3770)}{100}}^2\right)$ $\sim N(0.3770, 0.0485^2)$
✓ Correctly calculates $P(X \geq 6)$ ✓ Uses sample size to justify approximation by normal distribution ✓ Normal distribution used ✓ Correctly calculates the normal distribution parameters

Question 4 **(9 marks)**

A random sample of 600 people found that 144 of them owned dogs.

(a) Determine the sample proportion, $\hat{p}$, of those who owned dogs.

(1 mark)

$\hat{p} = \dfrac{144}{600} = 0.24$
✓ Correct value

(b) Use the survey results to estimate the standard deviation of $\hat{p}$.

(2 marks)

$\sigma_{\hat{p}} = \sqrt{\dfrac{0.24 \times 0.76}{600}} = 0.0174$
✓ Correct substitution into formula ✓ Correct answer

(c) Another survey is to be taken. Using a 99% confidence interval and the sample proportion from the initial survey, estimate the sample size required to ensure the margin of error is at most 0.04.

(4 marks)

$ME = k \times \sqrt{\dfrac{\hat{p} \times (1-\hat{p})}{n}}$ $k = invNormCDF(\text{"C"}, 0.99, 1, 0) = 2.576\ldots$ $0.04 = (2.576\ldots) \times \sqrt{\dfrac{0.24 \times 0.76}{n}}$ $n = 756.378\ldots$ ∴ a sample size of 757 is required
✓ Writes an equation to evaluate n from the margin of error ✓ Correct k value used ✓ Solves correctly for n ✓ Rounds final answer up to the nearest whole number

Question 4 (continued)

A 95% confidence interval for the population proportion is calculated from the original random sample and found to be $0.2058 \leq \hat{p} \leq 0.2742$.

(d) If a new sample of 360 people at the beach is taken and 135 are found to own dogs, what does this indicate?

(2 marks)

$\hat{p} = \frac{135}{360} = 0.375$ and $0.3250 \leq \hat{p} \leq 0.4250$ (Stat→Calc→Interval→One-Prop Z Int) $\hat{p}$ is well outside the original range and the confidence interval of the new sample is very different to the original interval so this indicates that the sample is likely to be biased.
✓ Calculates either $\hat{p}$ or 95% confidence interval correctly ✓ States the difference of results

Question 5 **(7 marks)**

A random sample of 150 investment bankers found that 65 had slept less than 30 hours that week as they were working so hard.

(a) Determine a 90% confidence interval for the population proportion p.

(2 marks)

$0.3668 \leq \hat{p} \leq 0.4999$ (Stat→Calc→Interval→One-Prop Z Int)
✓ Correctly calculates the lower value of the confidence interval ✓ Correctly calculates the upper value of the confidence interval

(b) If the true proportion of investment bankers who had slept less than 30 hours that week is 35%, what is the probability that the sample proportion is at most 0.4 in a new sample of 360 bankers?

(3 marks)

$\hat{p} \sim N\left(0.35, \left(\sqrt{\frac{0.35 \times 0.65}{360}}\right)^2\right)$ $\hat{p} \sim N(0.35, (0.02514)^2)$ $P(\hat{p} \leq 0.4) = 0.9766$
✓ Uses normal distribution ✓ States correct parameters for normal distribution ✓ Correctly calculates probability

(c) Another 11 surveys of investment bankers with sample size 100 were conducted and for each a 90% confidence interval for p was calculated. Calculate the probability that less than 8 of the intervals included the true value for p.

(2 marks)

$Let\ X\ be\ the\ number\ of\ samples\ out\ of\ the\ 11\ surveys\ that\ included\ the\ true\ value$ $X \sim Bin(11, 0.90)$ $P(X < 8) = P(X \leq 7) = 0.0185$
✓ Identifies distribution as binomial ✓ Correctly calculates probability

Question 6 **(5 marks)**

A business analyst for Wesfarmers wanted to evaluate the number of Target stores with 100% compliance in their marketing strategy. His survey of 150 store managers provided a confidence interval of $0.36 \leq p \leq 0.54$. Calculate the point estimate for the proportion of compliant stores in this sample and the level of confidence that the analyst can have in this interval.

$$\hat{p} = \frac{0.36 + 0.54}{2} = 0.45$$

$$ME = 0.54 - 0.45 = 0.09$$

$$ME = z \times \sqrt{\frac{0.45(0.55)}{150}} = 0.09$$

$$z = 2.21565\ldots$$

$$P(-2.21565 \leq z \leq 2.21565) = 0.9733$$

$\therefore$ the level of confidence is $\approx 97.3\%$

- ✓ Calculates sample proportion
- ✓ Calculates margin of error
- ✓ Correctly substitutes into margin of error equation
- ✓ Solves for z
- ✓ Calculates correct level of confidence

OR

$$\hat{p} = \frac{0.36 + 0.54}{2} = 0.45$$

$$\sigma = \sqrt{\frac{0.45(0.55)}{150}} = \sqrt{0.00165}$$

$$\hat{p} \sim N(0.45, 0.00165)$$

$$P(0.36 \leq \hat{p} \leq 0.54) = 0.9733$$

$\therefore$ the level of confidence is $\approx 97.3\%$

- ✓ Calculates sample proportion
- ✓ Calculates standard deviation
- ✓ Uses normal distribution
- ✓ Finds $P(0.36 \leq \hat{p} \leq 0.54)$
- ✓ Calculates correct level of confidence

End of Test